

# Finite-temperature resistive transition in the two-dimensional $XY$ gauge glass model

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We investigate numerically the resistive transition in the two-dimensional  $XY$  gauge glass model. The resistively-shunted junction dynamics subject to the fluctuating twist boundary condition is used and the linear resistances in the absence of an external current at various system sizes are computed. Through the use of the standard finite-size scaling method, the finite temperature resistive transition is found at  $k_B T_c = 0.22(2)$  (in units of the Josephson coupling strength) with dynamic critical exponent  $z = 2.0(1)$  and the static exponent  $\nu = 1.2(2)$ , in contrast to widely believed expectation of the zero-temperature transition. Comparisons with existing experiments and simulations are also made.

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Since the prediction of the vortex glass (VG) phase in the high- $T_c$  materials,<sup>1,2</sup> the properties and the existence of VG phase have drawn intensive interest for a decade.<sup>3–23</sup> In experiments<sup>3–7</sup> the VG phase has a significant practical importance since the high- $T_c$  materials can be truly superconducting in this phase where vortices are completely frozen at random positions, while the Abrikosov vortex lattice in pure type-II superconductors dissipate energy at any amount of external currents. To study VG from a theoretical point of view simplified discretized models such as the  $XY$  gauge glass and the  $XY$  spin glass have been widely used. For the gauge glass model, there is a growing consensus that the low critical dimension is less than 3, which means that there exists VG phase at finite temperatures in three dimensions (3D). This has been confirmed by the defect wall energy calculations at zero temperature<sup>8–11</sup> as well as by finite-temperature simulations.<sup>12–14</sup> Although the issue about the validity of using the gauge glass model to describe real bulk high- $T_c$  materials is not completely settled (e.g., the former is isotropic with a vanishing net external magnetic field, while the latter is anisotropic with nonzero net magnetic field), experiments on high- $T_c$  materials like YBCO<sup>3–5</sup> and BiSCCO<sup>6,7</sup> also have yielded results in accordance with the existence of a VG phase at finite temperatures.

In 2D, all existing defect wall energy calculations have unanimously revealed that the stiffness exponent has a negative value,<sup>9,11,15</sup> which has been confirmed in Ref. 16 through a simple argument but has been interpreted as an artifact of ubiquitous spin wave fluctuations. Finite-temperature simulations have so far yielded contradicting results: On the one hand, zero-temperature transition has been concluded from Monte-Carlo (MC) simulations in earlier studies<sup>13,15,17</sup> and from the current-voltage ( $IV$ ) characteristics in resistively-shunted junction (RSJ) simulations.<sup>18,19</sup> On the other hand, there have been evidences of the finite-temperature transition from the  $IV$  characteristics<sup>20</sup> and from the divergence of the relaxation time scale<sup>21</sup> in RSJ dynamic simulations, and from the very recent extensive finite-temperature

MC simulations.<sup>16</sup> Also, Langevin dynamic simulation of a vortex model with quenched impurities, which is closely related with the gauge glass model, also has found the finite-temperature transition in 2D.<sup>23</sup> To make things more complicated, absence of finite-temperature VG phase has been reported in early experiments on very thin YBCO films,<sup>4</sup> whereas a recent study on thin YBCO films, where the correlation length in  $c$ -direction exceeds the film thickness,<sup>5</sup> as well as experiments on highly isotropic BiSCCO [Bi(2:2:2:3)]<sup>7</sup> has obtained results which can be interpreted as indications of finite- $T$  VG phase in 2D. Consequently, we believe that the question about the existence of the finite- $T$  VG transition is not resolved yet. To our knowledge, a detailed numerical study of transport properties of 2D gauge glass model has not been performed: Existing RSJ studies<sup>18,19,21</sup> did not consider the finite-size scaling in a proper way and in some of them<sup>18,19</sup> the temperature range used did not cover the expected transition temperature ( $T_c \approx 0.22$  in Refs. 16 and 21, and  $T_c \approx 0.15$  in Ref. 20).

In this paper, we use RSJ dynamic simulation to investigate the resistive transition in the 2D  $XY$  gauge glass model in the absence of external currents. Since the voltage is always nonzero in the presence of a finite current because of the nucleation process, vanishing of the *linear* resistance, which is defined in the limit of zero external current, is the appropriate definition of a superconductor. The fluctuating twist boundary condition (FTBC) for Langevin-type dynamic simulation<sup>24</sup> made it possible to calculate the linear resistance with preserved periodicity of the phase variables (similar methods have also been used in Ref. 25). We here use FTBC to calculate the linear resistances, and then, through the standard finite-size scaling analysis (see, e.g., Refs. 2 and 26), a resistive transition at a finite temperature is concluded.

The Hamiltonian of the 2D  $L \times L$   $XY$  gauge glass model subject to FTBC is written as<sup>24,27,28</sup>

$$H = -J \sum_{\langle ij \rangle} \cos(\theta_i - \theta_j - \mathbf{r}_{ij} \cdot \mathbf{\Delta} - A_{ij}), \quad (1)$$

where  $J$  is the Josephson coupling strength, the summa-

tion is over all nearest neighboring bonds, and  $\theta_i$  (with periodicity  $\theta_i = \theta_{i+L\hat{x}} = \theta_{i+L\hat{y}}$ ) is the phase of the superconducting order parameter at site  $i$ . The twist variable  $\Delta = (\Delta_x, \Delta_y)$  measures the global twist of phase variables in each direction, and  $\mathbf{r}_{ij}(= \hat{x}, \hat{y})$  is the unit vector from site  $i$  to the nearest-neighbor site  $j$  (the lattice spacing is taken to be unity). The XY gauge glass model is characterized by the quenched random variable  $A_{ij}$  uniformly distributed in  $[-\pi, \pi)$ , whereas in the XY spin glass model  $A_{ij}$  can have binary values 0 and  $\pi$  with equal probability. Recently XY spin glass model draws attention in relation to the  $\pi$ -junctions due to the  $d$ -wave symmetry of order parameters.

The equations of motion for RSJ dynamics subject to FTBC are determined from the local and the global current conservations and are written in dimensionless forms (see Refs. 24 and 28 for details): For phase variables

$$\dot{\theta}_i = - \sum_j G_{ij} \sum_k' [\sin(\theta_j - \theta_k - \mathbf{r}_{jk} \cdot \Delta - A_{jk}) + \eta_{jk}], \quad (2)$$

where  $G_{ij}$  is the 2D square lattice Green's function and the primed summation is over four nearest neighbors  $k$  of site  $j$ , while for the twist variables

$$\dot{\Delta} = - \frac{1}{L^2} \frac{\partial H}{\partial \Delta} + \eta_{\Delta}. \quad (3)$$

The thermal noise terms satisfy  $\langle \eta_{ij} \rangle = \langle \eta_{\Delta} \rangle = \langle \eta_{ij} \eta_{\Delta} \rangle = \langle \eta_{\Delta_x} \eta_{\Delta_y} \rangle = 0$  and

$$\begin{aligned} \langle \eta_{ij}(t) \eta_{kl}(0) \rangle &= 2T \delta(t) (\delta_{ik} \delta_{jl} - \delta_{il} \delta_{jk}), \\ \langle \eta_{\Delta_x}(t) \eta_{\Delta_x}(0) \rangle &= \langle \eta_{\Delta_y}(t) \eta_{\Delta_y}(0) \rangle = \frac{2T}{L^2} \delta(t), \end{aligned}$$

where  $\langle \dots \rangle$  is the thermal average, and the time  $t$  and the temperature  $T$  have been normalized in units of  $\hbar^2/4e^2 R_0 J$  with the shunt resistance  $R_0$ , and  $J/k_B$ , respectively. The equations of motion were integrated numerically by using the efficient second-order Runge-Kutta-Helfand-Greenside algorithm<sup>29</sup> with the discrete time step  $\Delta t = 0.05$ . For each disorder realization we neglected an initial stage of the time evolution up to  $t_0$ , and the measurements were made between  $t_0$  and  $t_0 + t_m$ . For  $L = 4, 6, 8$ , and  $10$ , disorder averages were performed over  $N_s = 400, 200, 100, 50$  samples, respectively. We verified that  $t_0 = 10^6$  and  $t_m = 5 \times 10^6$  (corresponding to  $10^8$  time steps) were sufficiently large; it is more important to increase the number of sample averages to have more reliable results than increasing  $t_m$  further. The highest temperature  $T = 0.30$  in the present work corresponds to the lowest temperature in Ref. 18, and this makes it necessary to use much larger value of  $t_m$  than in Ref. 18 [by factor  $O(10^2)$ ].

The linear resistance in  $x$  direction is given by the Nyquist formula:<sup>30</sup>

$$R_x = \frac{1}{2T} \left[ \int_{-\infty}^{\infty} dt \langle V_x(t) V_x(0) \rangle \right] \quad (4)$$

with the disorder average  $[\dots]$  and the voltage across the whole sample  $V_x = -L\Delta_x$ , which is then approximated as<sup>31</sup>

$$R_x \approx \frac{L^2}{2T} \frac{1}{\Theta} [\langle [\Delta_x(\Theta) - \Delta_x(0)]^2 \rangle], \quad (5)$$

for sufficiently large  $\Theta$ . For convenience, we define

$$f(\Theta) \equiv \frac{L^2}{2T} \frac{[\langle [\Delta_x(\Theta) - \Delta_x(0)]^2 \rangle] + [\langle [\Delta_y(\Theta) - \Delta_y(0)]^2 \rangle]}{2} \quad (6)$$

and calculate the linear resistance  $R$  from the least-square fit to the form  $f(\Theta) = R\Theta$ .

We plot in Fig. 1 for  $L = 6$  at  $T = 0.22$  (a) the time evolution of  $L\Delta_y$  for a given disorder realization, and (b)  $f(\Theta)$  obtained from the average over 200 samples. The full line in Fig. 1(b) obtained from the fit determines the value of  $R$ . Figure 2 shows  $R$  obtained in this way as a function of  $L$  at  $T = 0.30, 0.25, 0.23, 0.22, 0.21, 0.20$ , and  $0.15$ . For each  $L$  and  $T$ , we divided the total number of samples,  $N_s$ , into five groups, each of which contains  $N_s/5$  samples, and  $R$  was calculated for each group, leading to the estimation of error bars (standard deviations) in Fig. 2. The linear resistance shows clear change of curvature around  $T \approx 0.22$ : In the high-temperature regime  $R$  appears to saturate as  $L$  is increased, while in the low-temperature regime  $R$  drops down more rapidly as  $L$  is increased. To see this behavior more clearly, we also display in the inset of Fig. 2  $R(T = 0.15)/R(T = 0.30)$  as a function of  $L$  in log scales, which does not exhibit any sign of saturation in terms of  $L$ : On the contrary, the downward curvature implies that in the thermodynamic limit  $R(T = 0.15)$  vanishes. These observations lead to the conclusion that the 2D XY gauge glass model has the *finite-temperature resistive transition*.

To obtain more precise value of  $T_c$ , we proceed to the finite-size scaling analysis of the linear resistance. In the dynamic scaling theory for the usual second-order phase transition in 2D,<sup>2</sup> the linear resistance has the standard finite-size scaling form:

$$R(L, T) = L^{-z} \rho \left( (T - T_c) L^{1/\nu} \right), \quad (7)$$

where  $\rho(x)$  is the scaling function with the scaling variable  $x$ ,  $z$  is the dynamic critical exponent, and the static critical exponent  $\nu$  is defined by the divergence of the correlation length, i.e.,  $\xi \sim |T - T_c|^{-\nu}$ . At  $T = T_c$ ,  $R \sim L^{-z}$  [see Eq. (7)] and from Fig. 2 we obtain  $z \approx 2.0$ . Although this value is much smaller than the values usually measured in experiments ( $z = 4 \sim 9$ ), it should be noted that the similar values have also been observed in various numerical simulations with RSJ dynamics and MC dynamics of 2D gauge glass model ( $z \approx 2.2$  in Ref. 20 and

$z \approx 2.4$  in Ref. 16, respectively) as well as the Langevin-type relaxational dynamics of vortices in the presence of quenched impurities ( $z \approx 2.1$ , Ref. 23). Furthermore, a recent analytic calculation based on the dynamic renormalization group method also has found  $z = 2$  for 2D gauge glass model with a purely relaxational equations of motion.<sup>22</sup> In contrast, widely believed zero-temperature transition in 2D gauge glass model implies that the linear resistance should have Arrhenius form and thus corresponds to  $z = \infty$ . We also tried to fit our data in Fig. 2 to the Arrhenius form<sup>18,19</sup> (see Fig. 3) and found that the thermal activation barrier, which is proportional to the slopes in Fig. 3, strongly depends on the system size. Furthermore,  $R$  is found to deviate from the Arrhenius form in a systematic way at the lowest temperature at all system sizes.

Figure 4 shows the scaling plot Eq. (7) with  $z = 2.0$ ,  $T_c = 0.22$ , and  $\nu = 1.2$ . We tried to vary parameter values used in the scaling plot and concluded

$$T_c = 0.22(2), \quad (8)$$

$$z = 2.0(1), \quad (9)$$

$$\nu = 1.2(2), \quad (10)$$

where numbers in parenthesis denote errors in the last digits, and  $T_c \approx 0.22$  is in a good agreement with Refs. 16 and 21. It is worth mentioning that the same method (calculation of the linear resistance with FTBC accompanied by the finite-size scaling analysis) has lead to a very precise determination of  $T_c$  in 3D XY model.<sup>28</sup> In recent studies of 2D XY gauge glass model based on the same RSJ dynamics,<sup>18,19</sup> the finite-size scaling analysis has not been used and the temperature range did not cover the critical temperature ( $T_c \approx 0.22$ ), leading to the different conclusion of the zero-temperature transition.

Recently, the possibility of the *quasi-long-range* glass order with the vanishing glass order parameter<sup>32</sup> in 2D gauge glass model has been suggested in Ref. 16, where it has been argued that the correlation length  $\xi$  diverges in the whole low-temperature phase. Although our scaling plot in Fig. 4 was obtained on the assumption of the usual second order transition, where  $\xi$  is finite both below and above  $T_c$ , it is still plausible to have the quasi-long-range glass order: In this case, the quasi-criticality should be reflected in the temperature-dependent dynamic critical exponent below  $T_c$  with  $R(L, T) \sim L^{-z(T)}$ , while the high-temperature phase still obeys the scaling form in Eq. (7). Our current results cannot rule out this possibility, and if we neglect the smallest size  $L = 4$  in Fig. 2,  $R$  in the low-temperature phase indeed appears to exhibit the simple algebraic form  $R \sim L^{-z}$  with increasing  $z$  as  $T$  is decreased, in accord with the idea of the quasi-long-range glass order.

In conclusion, we studied numerically the resistive transition in the 2D XY gauge glass model by using the RSJ dynamic equations subject to the fluctuating twist boundary condition. The standard finite-size scaling analysis applied to the linear resistances lead to the

strong evidence of the *finite-temperature* resistive transition at  $T_c = 0.22(2)$  with the dynamic critical exponent  $z = 2.0(1)$  and the static exponent  $\nu = 1.2(2)$ . However, the nature of this finite-temperature resistive transition needs to be investigated more in detail.

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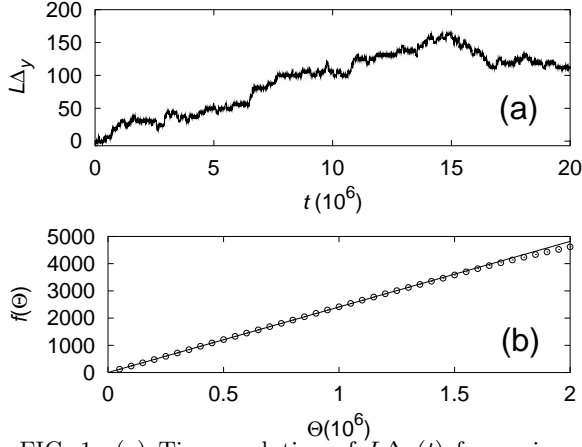


FIG. 1. (a) Time evolution of  $L\Delta_y(t)$  for a given disorder realization for  $L = 6$  at  $T = 0.22$ . (b)  $f(\Theta)$  versus  $\Theta$  [see Eq. (6)] obtained from 200 sample averages for  $L = 6$  at  $T = 0.22$  (open circles). Least-square fit to the form  $f(\Theta) = R\Theta$  (full line) determines the linear resistance.

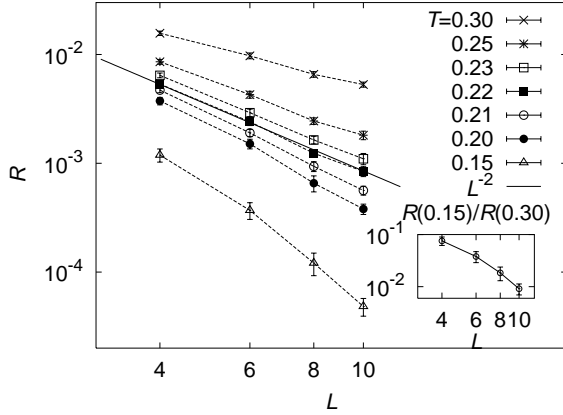


FIG. 2. Linear resistance  $R$  versus system size  $L$  at temperatures  $T = 0.30, 0.25, 0.23, 0.22, 0.21, 0.20$ , and  $0.15$  (from top to bottom). It is shown that the behavior in the high-temperature regime ( $T \gtrsim 0.25$ ) is different from that in the low-temperature regime ( $T \lesssim 0.15$ ), implying the existence of the resistive transition near  $T = 0.22$ , where the power-law behavior  $R \sim L^{-2}$  also determines the dynamic critical exponent  $z \approx 2$ . Inset: The ratio  $R(T = 0.15)/R(T = 0.30)$  versus  $L$ . It is exhibited that the ratio shows downward curvature as  $L$  is increased, which implies that  $R(T = 0.15)$  vanishes in the thermodynamic limit and that the resistive transition occurs at  $T_c > 0.15$ .

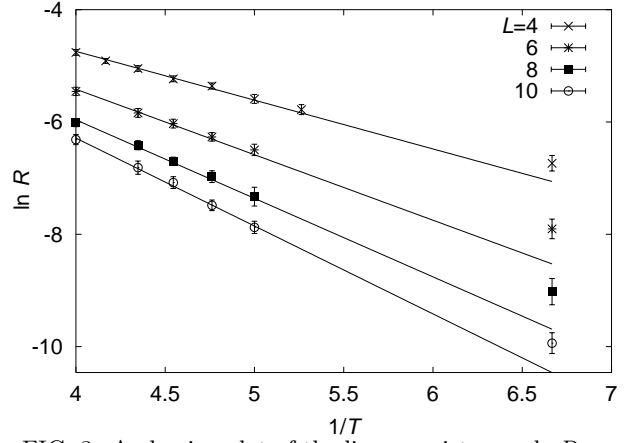


FIG. 3. Arrhenius plot of the linear resistance:  $\ln R$  versus  $1/T$ .  $R$  shows a systematic deviation at the lowest temperature for all system sizes used.

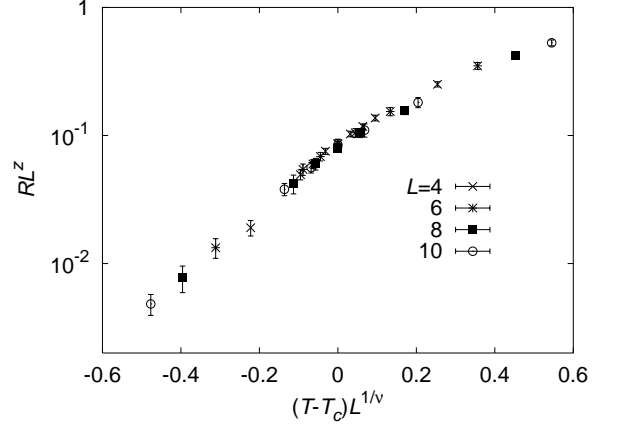


FIG. 4. Finite-size scaling plot of the linear resistance:  $RL^z$  versus the scaling variable  $(T - T_c)L^{1/\nu}$  with  $T_c = 0.22$ ,  $\nu = 1.2$ , and  $z = 2.0$ . (All data points in Fig. 2 have been used.)